Bsp. 1

$$\begin{aligned} -x_1 + 2x_2 + 6c\,x_3 &= 2 \\ 2x_1 - 3x_2 - 9c\,x_3 &= -4 \\ -x_1 + 3x_2 + 3c^2 x_3 &= 11 - c^2 \end{aligned}$$

$c \in \mathbb{R}$

Für welche c { eind. Lsg. / unendlich Lsg. / keine Lsg }

?

Bsp 2

$$\begin{aligned} kx + y + z &= 0 \\ x + ky - z &= 0 \\ x - y + kz &= 0 \end{aligned}$$

Für welche Werte von k ex. nichttriviale Lsgen?

$$\left(\begin{array}{ccc|c} k & 1 & 1 & 0 \\ 1 & k & -1 & 0 \\ 1 & -1 & k & 0 \end{array}\right) \begin{array}{l} \\ \mid(-k) \\ \mid(-k) \end{array} \quad (k \neq 0)$$

$$\left(\begin{array}{ccc|c} k & 1 & 1 & 0 \\ 0 & 1-k^2 & 1+k & 0 \\ 0 & k+1 & 1-k^2 & 0 \end{array}\right) \begin{array}{l} \\ \\ \mid -(1-k) \end{array}$$

$$\left(\begin{array}{ccc|c} k & 1 & 1 & 0 \\ 0 & 1-k^2 & 1+k & 0 \\ 0 & 0 & -k(k^2-k-2) & 0 \end{array}\right)$$

NR:

$$\begin{aligned} &-(1-k^2)(1-k) + 1 + k \\ &-(1-k-k^2+k^3) + 1 + k \\ &= \cancel{-1} + k + k^2 - k^3 \cancel{+1} + k \\ &= -k^3 + k^2 + 2k \\ &= -k(k^2 - k - 2) \end{aligned}$$

$$\begin{array}{ccc|c} k & 1 & 1 & 0 \\ 0 & 1-k^2 & 1+k & 0 \\ 0 & 0 & -k(k-2)(k+1) & 0 \end{array}$$

$k^2 - k - 2 = 0$ | 3

$k_{1/2} = \frac{1 \pm \sqrt{1+8}}{2}$

$k_1 = 2$

$k_2 = -1$

Nichttriviale Lsg

für $k = 2$ und $k = -1$

Für $k = 0$ spezielle Untersuchung:

$$\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 \end{array}$$

$$\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{array} \quad | (-1)$$

$$\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{array} \quad | (-1)$$

$$\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -2 & 0 \end{array}$$

$\Rightarrow z = 0 \Rightarrow y = 0 \Rightarrow x = 0$

$\Rightarrow$ einz. Lsg. für $k = 0$.

Lsg. zu Bsp. 1:

$$\left(\begin{array}{ccc|c} -1 & 2 & 6c & 2 \\ 2 & -3 & -9c & -4 \\ -1 & 3 & 3c^2 & 11-c^2 \end{array}\right) \quad \begin{matrix} \cdot 2 \\ \cdot(-1) \end{matrix}$$

$$\left(\begin{array}{ccc|c} -1 & 2 & 6c & 2 \\ 0 & 1 & 3c & 0 \\ 0 & 1 & 3c^2-6c & 9-c^2 \end{array}\right) \quad \cdot(-1)$$

$$\left(\begin{array}{ccc|c} -1 & 2 & 6c & 2 \\ 0 & 1 & 3c & 0 \\ 0 & 0 & 3c^2-9c & 9-c^2 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -1 & 2 & 6c & 2 \\ 0 & 1 & 3c & 0 \\ 0 & 0 & 3c(c-3) & (3-c)(3+c) \end{array}\right)$$

L.S = 0: $c = 0 \vee c = 3$

R.S = 0: $c = 3 \vee c = -3$

$\Rightarrow$ Unendlich viele Lsg. $\Leftrightarrow$ $\boxed{c = 3}$

$\Rightarrow$ keine Lsg. $\Leftrightarrow$ $c = 0$

$\Rightarrow$ Lind. Lsg. $\Leftrightarrow$ $c \in \mathbb{R} \setminus \{0; 3\}$